**Classical Paramagnet**

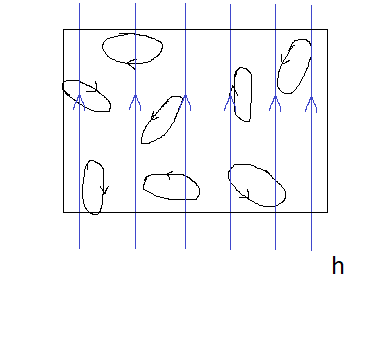
Now let’s look at some more examples… We *could* contrive to model a magnetic material as a bunch of randomly oriented current loops. This model finds support from the central field approximation to an atom, in a magnetic field. We’ll recall that in the last file we had for an atom (really a paramagnet) in a magnetic field (**h** is the bulk-interstitial field), ignoring the somewhat irrelevant HCFA:



which we could write as:



The magnitude of **J**T is fixed, and so **μ** is a vector with fixed magnitude, but according to Quantum Mechanics its component along the **h** direction is given by mjT = -JT, -JT + 1, -JT + 2, …, JT. So μ can basically change direction in the field. In this sense, the -**μ**·**h** term resembles the formula for the work required to rotate a fixed magnetic dipole / current loop in a magnetic field, and so that’s why it’s often classically approximated this way.



And so our energy levels are:



Now for large JT, the m’s can occupy a large range of values, and in a certain conceived limit, relatively continuous. And so we can/will approximate the partition function with an integral. Goes like this: the quantum mechanical sum would be something like:



and we can change variables to x = mjT/JT,



Then can see that spacing between points in the sum is 1/JT. As we take the large JT limit, these become differentially small. And so we can say,



And then if we say x = cosθ, we’ll have:



which we can write as:



Again, this is the same as what we would’ve presumed had we simply treated the Hamiltonian as being given simply by the work required to rotate a current loop, which can assume any orientation, with magnetic moment IA = μ, in a field h: H = -**μ**·**h** = -μhcosθ. Only thing we might not have guessed is that classically, the measure of the integral is d(cosθ) = -sinθdθ, not just dθ. Anyway, proceeding, we can calculate the partition function for one of these:



The associated Helmholtz free energy is:



In the low T (high β) limit, this will be:



In the high T (low β) limit, this would be:



Now say we want the average magnetic moment, **M**atom, along the field direction. So statistically, this would be:



where F is the Helmholtz free energy. This result is expected of course, as it follows from the Thermodynamics/Equilibrium Systems file result M = -∂E/∂Bf, where in that context Bf is the external field, which would be the role h plays here. So proceeding, we have:



Now we can define the Langevin function:



Note the following limits,



So it starts off at zero and is linear, and asymptotes to 1. In any event, we can write the magnification as:



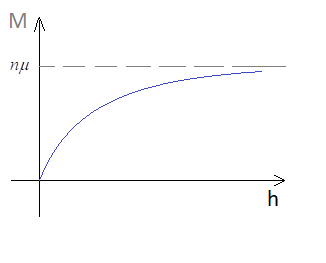
In the high T regime, βμh is small, and we may say:



This makes the magnetization **M** = **M**atom(N/ΔV), where N = # atoms, and ΔV the total volume, to be:



The proportionality with h is known as, well, Curie’s law. Note h is the *total* magnetic field, not the free field, Bf (= μ0H for homogeneous substance in solenoidal geometry, say), and so the constant of proportionality wouldn’t technically be the susceptibility *proper* I don’t think. The general plot of **M** vs. h would go like this:



We can calculate the entropy:



For small T’s (high β) this would be:



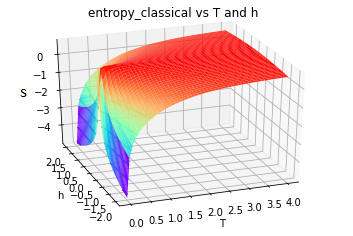
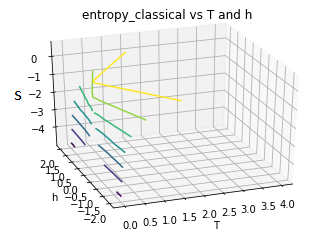
This doesn’t go to 0 for small T. And that’s because we’re doing a classical calculation, which is not valid at small T. For large T (small β),



Might note this maxes out for high T, which makes sense since there is a limit to how random the dipoles can get. The heat capacity would be, for large T:



Anyway, the exact function for S shows something interesting, that S is just a function of βh (μ being just some constant), so S = S(βh). Now Matom as well is a function of βh, so Matom = Matom(βh). From this we can conclude that Matom is a function of entropy alone: Matom = Matom(S). This conclusion will hold in the quantum case too, and will be implicitly used in the Bragg-Williams theory of ferromagnetism – in Mean Field Theory files. But this means that when we adiabatically (constant S, i.e., no heat transfer) increase the field, the magnetization must not increase at all. I guess what happens is that as we adiabatically increase the field strength, the temperature must also increase, somehow. And so the propensity of the stronger field to align the dipoles is canceled by the propensity of the higher temperature to randomize the dipoles. Conversely, if we adiabatically decrease the field, the magnetization won’t change, but the magnet should then cool. Made a graph. One on right is a contour plot. I just wanted to illustrate that adiabatic demagnetization (entropy constant, field decreasing) will cool the paramagnet. It’s more pronounced for a ferromagnet (see mean field theory files).

So yeah it’s easiest to see in the right plot perhaps. Starting from h = 0 and increasing |h|, while keeping S constant, we see that T increases. Well I guess that’s adiabatic *magnetization* → temperature *increase*. Same thing. This is given a few heuristic explanations. One is that as we increase the external field, it more and more aligns the dipoles with the field. And so their configurational entropy decreases. Could say their range of motion decreases. But to keep their entropy constant (because we’re doing this adiabatically), we must therefore have the Temperature go up, which I guess you could take to mean that the dipoles are now oscillating about their mean position more rapidly. This is entirely like how adiabatically compressing an ideal gas reduces its configurational space, and also increases its temperature (because it does work on it). But problem with our explanation is that when we place a paramagnet in an external field and increase h, the field actually does *negative* work on the paramagnet – see EM folder. But…the internal energy is decreasing even more, so, measured from the base, the energy imparted by the field is positive in a sense. Anyway, let’s work out the energy. This is:



where Matom is the thermal expectation of the atom’s magnetic moment of course. Can put this in terms of entropy. Solving for T in terms of S:



Then,



Checking out work-energy equation (see Thermodynamics/Equilibrium Systems, at constant entropy:



So that checks out. Good!

**Example**

A dipole-dipole interaction interaction is given by:



where θ is the angle between the two dipoles. What is the thermally averaged interaction energy? Well we did this already. But … the probability density for being at angle θ is:



The normalization factor is:



So the thermally averaged interaction is:



So let’s calculate Z,



So now,



Again, if we define the Langevin function,



then we can write this as:



**Classical Material in a Magnetic Field**

Despite the paramagnet-as-current loop-in-field analogy, it futile to attempt an *actual* model under *classical* presuppositions (note all the work we did above presupposed a quantum model which we then implicitly took the high JT ~ high temperature limit of). For consider a bunch of particles confined to a box under the influence of an electromagnetic field, and interacting via some pair-wise potential energy, say, V(xi) [a function of all the coordinates]. Then the partition function is as follows.



where in last line we simply make a change of variables. And so we see no contribution from the field. This makes sense because in our classical dipole model above, where we pretended the Hamiltonian was simply (negative) the work the magnetic field does when the dipole is rotated in it, we neglected the *back emf* work on the current itself which the magnetic field would *also* do, and which would be equal and opposite to the torque work. Therefore, the net work done by the magnetic field would be zero (work done by B fields is always zero), and so it wouldn’t affect the energy at all, and so shouldn’t show up in our partition function. So our result here illustrates that in classical models, there can be no magnetic susceptibility, and evinces a need for quantum mechanics to explain these properties.